

ASSIGNMENT XII RELATIONS AND FUNCTIONS

1. Let $*$ be a binary operation defined by $a * b = 3a + 4b - 2$. Find $4 * 5$.
2. Show that the relation R defined by $R = \{ (a, b) : a - b \text{ is divisible by } 3, a, b \in \mathbb{N} \}$ is an equivalence relation.
3. If $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{3x+5}{2}$ is an invertible function. Find f^{-1} .
4. If the function $f : \mathbb{R} \rightarrow \mathbb{R}$ is given by $f(x) = x^2 + 3x + 1$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ is given by $g(x) = 2x - 3$. Find $f \circ g$ and $g \circ f$.
5. If $f(x) = 27x^3$ and $g(x) = x^{1/3}$. Find $g \circ f$.
6. Consider $f : \mathbb{R}_+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible .and also find f^{-1} .
7. Let $A = \mathbb{N} \times \mathbb{N}$ and $*$ be a binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$. Show that $*$ is commutative, associative. Also find the identity element for $*$ on A , if any.
8. Show that the relation S defined on the set $\mathbb{N} \times \mathbb{N}$ by $(a, b) S (c, d) \iff a + d = b + c$ is an equivalence relation.
9. State the reason for the relation R in the set $\{1, 2, 3\}$ given by $R = \{ (1, 2), (2, 1) \}$ not to be transitive.
10. Consider the binary operation $*$ on the set $\{1, 2, 3, 4, 5\}$ defined by $a * b = \text{minimum of } a \text{ and } b$. write the operation table for $*$.
11. Prove that the relation R in the set $\{5, 6, 7, 8, 9\}$ given by $R = \{ (a, b) : |a - b| \text{ is divisible by } 2 \}$ is an equivalence relation. Find all elements related to the element 6.
12. Let $f : \mathbb{W} \rightarrow \mathbb{W}$ be defined as $f(x) = x - 1$ if x is odd, and $f(x) = x + 1$ if x is even. Show that f is invertible. Find the inverse of f .
13. Let \mathbb{N} denote the set of all natural numbers and R be the relation on $\mathbb{N} \times \mathbb{N}$ defined by $(a, b) R (c, d) \iff ad (b + c) = bc (a + d)$. Show that r is an equivalence relation.
14. If $f : \mathbb{R}_+ \rightarrow [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible .and also find f^{-1} .
15. If $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3x + 2$. Find $f(f(x))$.

